Non-idealities in the $3\omega$ method for thermal characterization in the low- and high-frequency regimes

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This work is devoted to analytical and numerical studies of diffusive heat conduction in configurations considered in $3\omega$ experiments, which aim at measuring thermal conductivity of materials. The widespread 2D analytical model considers infinite media and translational invariance, a situation which cannot be met in practice in numerous cases due to the constraints in low-dimensional materials and systems. We investigate how thermal boundary resistance between heating wire and sample, native oxide and heating wire shape affect the temperature fields. 3D finite element modelling is also performed to account for the effect of the bonding pads and the 3D heat spreading down to a typical package. Emphasis is given on the low-frequency regime, which is less known than the so-called slope regime. These results will serve as guides for the design of ideal experiments where the 2D model can be applied and for the analyses of non-ideal ones. © 2018 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/). https://doi.org/10.1063/1.5027396

I. INTRODUCTION

The $3\omega$ method\textsuperscript{1–11} has been proposed in 1910 by O. Corbino to measure the thermal diffusivity of metal filaments for use in light bulbs.\textsuperscript{12,13} In 1987, it was used by Birge and Nagel to measure frequency-dependent specific heat of a liquid.\textsuperscript{14} It was later\textsuperscript{1} that Cahill used the $3\omega$ method to measure the thermal conductivity of dielectric solid film. Although it was initially developed to measure the thermal conductivity of bulk materials, the $3\omega$ method was extended to the measurement of thin films down to 20 nm thick.\textsuperscript{15} The $3\omega$ method was then adapted to measure the in-plane and cross-plane thermal conductivities of anisotropic films,\textsuperscript{16,17} as well as the thermal properties of a variety of samples, such as nanowires and carbon nanotubes,\textsuperscript{18,19} liquids and gases,\textsuperscript{24} and free-standing membranes.\textsuperscript{25} One of the main advantages of the $3\omega$ method is that it is based on electro-thermal heating, which is well mastered, and that an analytical model based on 2D heat diffusion allows to determine the thermal conductivity of the sample in a very simple manner. While such model is very efficient to describe materials with at least two dimensions larger than 100 $\mu$m, it happens that it is insufficient for many of the configurations involving nano-objects or even Micro-ElectroMechanical Systems (MEMS), where space constraints do not allow to perform the measurements in the ideal configurations. The present article has the goal to analyze the effects of some deviations to the ideal configurations and serve as guide if one wants to analyze these effects. This paper is devoted to analytical and numerical studies of diffusive heat conduction depending on the heater size on top of flat substrate and on the geometry. It complements previous papers (see for instance Refs. 2,3,20–23) which dealt mostly with analytical expressions or could not account for the full environment of the heater. First we remind the transient regime for a line heat source and then we study it in the case of the $3\omega$ method. In particular, attention is paid to the low-frequency regime, which is more adapted to

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small dimensions. In Section III, the influences of thermal boundary resistances and thin insulating oxide layers between the heating wires and the samples are respectively assessed. The impact of the heater thickness on thermal conductance $G_{th}$ is also analyzed. It is found that the aspect ratio \{heater width over thickness\} can impact the temperature field. All these effects can be analyzed within the two-dimensional framework. For accurate characterization of diffusive heat conduction, 2D and 3D numerical simulations are then performed and compared. In Section IV, the effect of three-dimensional heat spreading is first investigated. Then the impact of the connections to the electrical circuit are studied. Finally, the influence of the packaging on the thermal conductance is scrutinized. All these results show that many deviations to the 2D model can be present in the experiments and they can be accounted for if one wants an accurate analysis of the experiment. In this paper results are given in terms of thermal conductance, which is proportional to the thermal conductivity in the 2D ideal models, or in terms of ratio of thermal conductances. As such, the results allow determining precisely the error committed by using the 2D semi-infinite model (high-frequency case) or the 2D finite medium model with fixed-temperature condition (low-frequency case).

II. PRINCIPLE AND ANALYTICAL SOLUTIONS

A. Reminder of the principle

For the sake of completeness, a detailed reminder of the principle of the $3\omega$ method is given in the following. A generic schematic of the experimental configuration in the $3\omega$ method is presented in Fig. 1. An AC driving current of angular frequency $\omega = 2\pi f$ is applied through the two outer pads of the device

$$I(t) = I_0 \cos(\omega t),$$

where $I_0$ is the current amplitude passing through the heater. The metallic wire undergoes Joule heating with power $P(t) = RI(t)^2$, which can be expanded as follows:

$$P = \frac{1}{2} I_0^2 R_0(1 + \cos(2\omega t)).$$

The power dissipated through the heater is therefore composed of two components: $P_{AC} = \frac{1}{2} I_0^2 R_0 \cos(2\omega t)$ is the component that depends on the second harmonic oscillation $2\omega$ and the constant component is $P_{DC} = \frac{1}{2} I_0^2 R_0$. The corresponding temperature in the substrate is also composed of two components, a DC one and an AC ($2\omega$) component. The temperature rise of the heater in response to the excitation is expressed as:

$$\Delta T(t) = T(t) - T_0 = \theta_{DC} + |\theta_{2\omega}| \cos(2\omega t + \varphi_{2\omega}),$$

where $|\theta_{2\omega}|$ is the amplitude of the temperature rise corresponding to AC component of power $P_{AC}(t)$, $\theta_{DC}$ is the steady state temperature increase due to time independent component of power $P_{DC}$ and

FIG. 1. Deposited metallic wire on top of substrate: heater/sensor in the $3\omega$ method.
where $\varphi_{2\omega}$ is a phase due to possible lag between temperature and the flux. $T_0$ is the ambient temperature. The electrical resistance of the metallic line due to temperature increase can be written following the Bloch-Grüneisen formula:

$$R(t) = R_0 (1 + \alpha \Delta T(t))$$

(5)

$$= R_0 + R_0 \alpha \theta_{DC} + R_0 \alpha |\theta_{2\omega}| \cos(2\omega t + \varphi_{2\omega}).$$

(6)

Consequently, the resistance is modulated at the second harmonic frequency. From Ohm’s law, the voltage drop across the heater results from the multiplication of the heater resistance (Eq. (6)) by the input current (Eq. (1)) yielding

$$V(t) = R_0 I_0 (1 + \alpha \theta_{DC}) \cos(\omega t) + \frac{\alpha |\theta_{2\omega}| R_0 I_0 \cos(\omega t + \varphi_{2\omega})}{2}$$

$$+ \frac{\alpha |\theta_{2\omega}| R_0 I_0 \cos(3\omega t + \varphi_{2\omega})}{2}.$$

Measuring the voltage signal at $3\omega$ can be challenging because it is usually smaller by three order of magnitude of the first voltage signal $1\omega$ due to the low value of the temperature coefficient of the electrical resistivity (TCR) $\alpha$ (in the range $\approx 10^{-3}$ K). However, the voltage at $3\omega$ is directly proportional to the temperature oscillation at $2\omega$: it is a direct thermometer of $\theta_{2\omega}$. The voltage $V_{3\omega}$ of the wire can be measured by using a lock-in amplifier (LIA) set-up, which can also be inserted in a Wheatstone bridge to remove the large $V_{1\omega}$ before measuring the third harmonic voltage signal $3\omega$. Consequently, the third harmonic component leads us to the values of $\theta_{2\omega}$ and $\varphi_{2\omega}$ as

$$|V_{3\omega}| = \frac{1}{2} |V_0| |\theta_{2\omega}| \quad \text{and} \quad \arg(V_{3\omega}) = \varphi_{2\omega},$$

(7)

where $V_0 = R_0 I_0$ is close to the peak amplitude of the voltage in the metallic line at first harmonic. We recall that both the third harmonic voltage $V_{3\omega}$ and the temperature oscillation temperature $\theta_{2\omega}$ are composed of an in–phase (real) and an out–of–phase (imaginary) components. By rearranging $V_{3\omega}$, the temperature amplitude of the heater can be measured as:

$$|\theta_{2\omega}| = \frac{2|V_{3\omega}|}{\alpha R_0 I_0} \approx \frac{2|V_{3\omega}|}{\alpha |V_{1\omega}|}.$$  

(8)

The analysis of the change of temperature amplitude with respect to frequency and the phase provides clues about the thermal properties of the sample on which the heating wire is lying on. Indeed, if the sample is thermally conductive, the wire is not heated efficiently since the thermal flux spreads into the sample. Conversely, if the sample is thermally insulating, the temperature of the wire increases because heat cannot be dissipated efficiently into the sample.

**B. Analytical solution for the temperature field**

A complete solution of the oscillating temperature field, through which the thermal conductivity of the sample can be found based on 2D heat diffusion, can be determined in various cases, such as the semi-infinite medium, or the finite substrate thickness with infinite lateral size. A general semi-analytical expression was given by Borca-Tasciuc et al.\textsuperscript{2} for multilayers (see also Refs. 22 and 23).

**1. Semi-infinite medium**

According to Cahill, the magnitude of the temperature oscillation for a finite width heater deposited on the surface of the substrate is:\textsuperscript{1}

$$\theta_{ac} = \frac{P}{\pi L \kappa} \int_0^\infty \frac{\sin^2(\lambda b)}{(\lambda b)^2 \sqrt{\lambda^2 + q^2}} d\lambda,$$

(9)

where $\frac{P}{L}$, $\kappa$, and $b$ are the input power per unit length (Wm\textsuperscript{-1}), thermal conductivity of the sample under study and the half width of the metallic line, respectively. In Eq. (9), $q$ is the wavenumber of the thermal wave and is given by $q^2 = \frac{2i\omega}{D}$. The real part of the reciprocal factor of the wave vector $q$ defines the thermal wavelength and the imaginary part is referred to as the penetration depth of the
heat wave. It is a measure of the depth at which the thermal wave penetrates into the substrate and writes

\[ \frac{1}{\text{Im}(q)} = \sqrt{D/\omega}. \] (10)

It is possible to solve Eq. (9) numerically with Mathematica. The integration is carried out for \( \lambda \) varying between \( 10^{-10} \) and \( 10^{10} \) m\(^{-1}\). Fig. 2 shows the real and imaginary components of the temperature oscillations as a function of excitation frequency. It exhibits a linear regime at low frequencies and a regime with some leveling-off at high frequencies. At low frequencies the linear regime can be characterized by a constant negative out-of-phase temperature oscillation and by an in-phase temperature oscillation decaying linearly with respect to the logarithm of the excitation frequency.

Analytical expressions can also be obtained. We note that in Eq. (9), the integrand decreases very quickly as a function of \( \lambda \) because the squared cardinal sine function \( (\sin(x)/x)^2 \) decreases quickly while it is multiplied by another decaying function. For low values of \( \lambda \),

\[ \lim_{b\lambda \to 0} \frac{\sin(b\lambda)}{b\lambda} = 1. \] (11)

We now restrict ourselves to the linear regime where the metallic line half width is much smaller than the thermal penetration depth, i.e. \( b < < 1/|q| \). In this regime \( 1/b > > |q| \) and as \( \lambda \) takes values up to \( 1/b \), the integrand behaves as \( 1/\lambda \) provided that \( \lambda \) is larger than \( |q| \). We find that \( \theta \) will behave logarithmically. It was shown numerically that the integral of Eq. (9) can be approximated by

\[ \bar{\theta}_{ac}(2\omega) \approx \frac{P}{\pi Lk} \int_{0}^{1/b} \frac{1}{\sqrt{\lambda^2 + q^2}} d\lambda \approx -\frac{P}{\pi Lk} \left( \ln(qb) + \xi \right). \] (12)

The constant \( \xi \) was found numerically to be 0.923.\(^{15,26}\) Its analytical value was provided by Duquesne et al., who found that \( \xi = 3/2 - \gamma \), where \( \gamma \) is the Euler constant. Substituting \( q = \sqrt{\omega^2/D} \) in Eq. (12) gives a relation between the temperature oscillation magnitude and the excitation frequency \( 2\omega \):

\[ \bar{\theta}_{ac}(2\omega) = -\frac{P}{2\pi Lk} \left[ \ln(2\omega) + \frac{1}{2} \ln\left( \frac{b^2}{D} \right) - 2\xi \right] - \frac{i P}{4Lk}. \] (13)

It is composed of two parts, the real part where the in-phase temperature oscillation decays logarithmically with respect to the excitation frequency \( 2\omega \) and the constant negative imaginary part of the out-of-phase temperature oscillations. Thermal conductivity of the substrate can be determined either from the real part or the imaginary part. But experimentally, it is more reliable to consider the data

![Fig. 2. In-phase and out-of-phase components of the temperature oscillations vs thermal excitation frequency 2\( \omega \) calculated using Eq. (9) using a thermal conductivity \( \kappa = 1 \) Wm\(^{-1}\)K\(^{-1}\), thermal diffusivity \( D = 1 \) m\(^2\)s\(^{-1}\), power per unit length \( P/L = 1 \) W/m, and line width \( 2b = 20 \) \( \mu \)m.](image)
from the real part contribution.\textsuperscript{1} To do so, Eq. (13) can be written in terms of measurable quantities by substituting Eq. (13) into Eq. (7). The third harmonic voltage $V_{3\omega}$ is given by

$$V_{3\omega} = -\frac{V_0^3 \alpha}{4\pi L \kappa R_0} \left[ \ln(2\omega) + \frac{1}{2} \ln \left( \frac{b^2}{D} \right) - 2\xi \right] - i \frac{V_0^3 \alpha}{8\pi L \kappa R_0}. \quad (14)$$

The linear relation between the third harmonic voltage and the logarithm of the excitation frequency $2\omega$ allows to calculate the slope, which is

$$\text{slope} = \frac{d(\text{Re}(V_{3\omega}))}{d(\ln(2\omega))} = \frac{V_0^3 \alpha}{4\pi L \kappa R_0}. \quad (15)$$

Hence, thermal conductivity of the substrate $\kappa$ can be calculated through the equation:

$$\kappa = \frac{V_0^3 \alpha}{4\pi L R_0} \times \frac{1}{\text{slope}}. \quad (16)$$

2. Finite medium

We now consider a medium of finite thickness $d_S$. If the frequency is low enough, the thermal penetration depth is much larger and the medium behaves as in the static regime (see Fig. 3). We consider a boundary condition with fixed temperature at the bottom of the medium. In this case, the temperature rise of the heater located on top of the finite substrate can be expressed as a function of the DC dissipation power ($P_{DC}$):

$$\Delta T = -\frac{P_{DC}}{\pi L \kappa} \left[ \ln \left( \frac{d_S}{b} \right) + \beta \right], \quad (17)$$

where $\beta$ is obtained semi-analytically and found here to be equal to 1.0484 for small ratios \{line width over medium thickness\}. The temperature rise of the heater depends on the logarithm of the heater width and the substrate thickness. For a given electrical power, the temperature amplitude increases when the heater width becomes narrower.

C. Thermal conductance in non-ideal cases and thermal conductivity

The temperature fields determined previously in the 2D high-frequency (equivalent semi-infinite medium) and low-frequency (equivalent finite medium) regimes are always inversely proportional to thermal conductivity (see Eqs. (13) and (17)). Temperature and power are related by a thermal conductance $G$ defined as $\theta_{ac} = P_{ac}/G(2\omega)$ and it can be noted that it is the proportionality factor between thermal conductance and thermal conductivity which changes when varying frequency:

![Fig. 3. In-phase component of the temperature oscillation vs thermal excitation frequency $2\omega$ for a finite silicon medium. Power input is $P/L = 19 \text{ mW}/256 \mu\text{m}$, width is $2b = 2 \mu\text{m}$, and thickness $d_S = 300 \mu\text{m}$.]
In fact, this parameter $K$ also depends on other inputs of the considered configuration, i.e., geometrical sizes if the medium cannot be considered infinite (see e.g., Eq. (17)), thermal conductivities of surrounding and interfacial materials, and thermal boundary resistances (TBR). In the following, we will compare the ideal cases ($K$ given by Eqs. (13) or (17)) to real cases, where the error committed on thermal conductivity $\kappa$ is due to erroneous estimation of $K$ with an ideal model. As a result, the error committed considering Eqs. (13) and (17) to determine a sample thermal conductivity from measurements is the relative difference between the real thermal conductance (the one measured, or simulated here) and the one of the idealistic analytical model. This relative difference can be quickly estimated from plots of the temperature fields or thermal conductances shown in the following.

III. TWO-DIMENSIONAL EFFECTS

A. Impact of the heater thickness

The 2D model presented previously considers the heater width but not its thickness. In reality, there should be some conditions between the thickness and the width of the heating wire. This issue was studied in particular analytically by Gurrum et al.\textsuperscript{20} and Wang and Sen.\textsuperscript{21} Emphasis was put on the validity of the slope regime approximation as functions of the ratios of thermal conductivity $\kappa_{\text{heater}}/\kappa_{\text{substrate}}$ and $h/2b$, where $h$ is the heater thickness. Here, the asymptotic case at low frequency is analyzed and the impact on the maximal and average temperatures in the wires is highlighted. A DC 2D FEM analysis is performed to study the impact of the heater thickness on the temperature at the surface of the substrate. A gold heater of 2 $\mu$m width, typical of many experiments where the wire is fabricated by photolithography, and of different thicknesses (see Fig. 4(a) for the temperature fields in the cross sections) is considered on top of a silicon substrate of thickness of 300 $\mu$m. We study the impact of the heater thickness on the thermal conductance by considering the maximum and averaging temperatures in the wire. The results are presented in Fig. 4(b), where it is observed that the heater thickness affects both the maximum and average temperatures of the wires. Experimentally, only the average temperature can be accessed. For a wire of 2 $\mu$m width, it is seen that the thermal conductance decays for thicknesses larger than 100 nm. Therefore, the influence of the thickness should be taken into consideration for a width/thickness aspect ratio 20. The fact that these maximal and average temperatures never coincide already induces an uncertainty with respect to a model considering an isothermal heater, here found to be 2.8% for an aspect ratio of 20.

B. Effect of an insulating layer below the heating wire

We now investigate the effect of the presence of a nanometre-thick electrically-insulating layer below the heating wire. We consider a thin film of oxide layer of thicknesses $d_{\text{SiO}_2} = 2$ nm, which is representative of a possible native oxide, and 15 nm on top of a finite Si substrate. The second case is studied for the purpose of analysing the influence of thin layers deposited below the heating wire to electrically isolate the sample under investigation when it is electrically-conducting. The temperature drop across the film is given by the difference in the temperature rise of the same heater under the same power dissipation conditions. The effect of the film can be described as a 1D thermal resistance in the cross–plane direction if the heater width is much larger than the film thickness. Note that the thermal conductivity of the oxide layer is often smaller than the substrate thermal conductivity ($\kappa_F < \kappa_S$). The oxide layer can be considered as an added thermal resistance $R_F = \frac{d_F}{\kappa_F}$, where

$$\Delta T_F = \frac{P}{2bL} R_F = \frac{P}{2bL} \frac{d_F}{\kappa_F},$$  \hspace{2cm} (18)

is the temperature decrease in the film, a real value independent of frequency. As a result, the temperature rise across the thin film and the studied substrate can be expressed as\textsuperscript{15}

$$T_{F+S} \approx \Delta T_F + \Delta T_S$$  \hspace{2cm} (19)

where $T_{F+S}$ is the heater temperature. Note that this expression is valid both in the low-frequency and high-frequency regimes provided that the thermal penetration depth is much larger than the film thickness. The effect of the two different film thicknesses with respect to different wire widths is
FIG. 4. Impact of heater thickness on temperature field in the wire and in the samples for a given width of 2 μm. (a) Temperature fields for various thicknesses of 0 μm, 0.5 μm, 2 μm and 3 μm. The samples are 300 μm thick and the computational domain is 1 mm large. (b) Thermal conductance as a function of wire thickness. $G_{th}$ is computed with average (red) and maximum (blue) temperatures.

Temperature rise is calculated in the static regime based on Eqs. (17) and (18). Oxide layers affect strongly the temperature rise of the heater. Fig. 5 indicates that nanowires are affected even more strongly. Such an oxide layer would multiply the temperature rise by a factor of 5 for a wire width of 100 nm. For usual native oxide layers on the order of 2 nm, the increase for a wire of 100 nm width is on the order of 40%. It is therefore important to consider this increase if the heating wires are narrow. This means that a minimum width should be recommended for the $3\omega$ method. Note that the results are provided in the static regime, and that the effect will be stronger at high frequency because the temperature rise is then smaller. For instance, the error related to the possible presence of a native oxide (in the diffusive regime) is limited to a maximum of 5% in the static regime if the width is larger than 2 micrometers.
FIG. 5. Effect of 2 nm (blue) and 15 nm (green) of SiO$_2$ on temperature rise of the heater. The films are located on top of Si substrate of 300 µm thickness. Electrical power source is 85 mW for a length of 256 µm.

C. Impact of thermal boundary resistance

It has long been questioned if thermal boundary resistance (TBR) can affect the temperature field below the wire, and therefore also inside it. To analyse the possible effect, we consider that a TBR exists at the interface between the heat source (heater) and the studied substrate.  

Thermal boundary resistance is often defined by its inverse, the thermal boundary conductance $h_k$. Typical thermal boundary resistances at metal–metal and metal–Si interfaces are given in Table I. While the effects could be observed in a 2D model, the results of this section have been obtained by means of a full 3D calculation in the static regime, which involves a realistic wire made both of gold and of an adhesion layer of chromium (10 nm) below the gold and on top of a silicon sample (see Section IV for more details on the 3D FEM simulation). Here, the TBR is considered at the interface of chromium and silicon. Fig. 6 shows the effect of TBR on the total thermal conductance of the studied substrate. For TBR < $10^{-3}$ MW$^{-1}$.m$^2$.K, the ratio of thermal conductance equals one for a wire width of 100 nm. In this case, the impact of interfacial boundary resistance on thermal transport is negligible. For values larger than $10^{-3}$ MW$^{-1}$.m$^2$.K, the ratio can increase to 125 for a TBR equal to 10 MW$^{-1}$.m$^2$.K. In this case, transmissivity is very weak and interfacial boundary resistance can impact strongly the thermal transport at the interface. For a wire width of 10 µm, the effect of the TBR is smaller. For TBR < 0.01 MW$^{-1}$.m$^2$.K, the ratio of thermal conductance still equals one. For values larger than 0.01 MW$^{-1}$.m$^2$.K, the ratio can increase by three orders of magnitude and thus interfacial boundary resistance can impact strongly the thermal transport at interface. The values provided by Table I discard an effect of TBR for the widths considered in Fig. 6 (10 µm and 100 nm). For narrower sources, the effect is stronger. The effect is parallel to the one of thin oxide layers.

IV. 3D EFFECTS

A. 3D heat dissipation and effects of the pads

To analyse the deviation to the ideal 2D model, a 3D FEM simulation is performed (in addition to 2D FEM simulation) which includes also the connecting pads at the sides of the heating wire. In order

<table>
<thead>
<tr>
<th>Contact layer</th>
<th>Mixed layer thickness [nm]</th>
<th>TBR [MW$^{-1}$.m$^2$.K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al/Cu</td>
<td>1</td>
<td>$2.22 \times 10^{-4}$</td>
</tr>
<tr>
<td>Al/Si</td>
<td>10</td>
<td>$2.63 \times 10^{-3}$</td>
</tr>
<tr>
<td>Cr/Si</td>
<td>8</td>
<td>$3.33 \times 10^{-3}$</td>
</tr>
</tbody>
</table>
FIG. 6. Impact of thermal boundary resistance $1/h_k$ on thermal conductance $G_{th}$ for a heater of widths 100 nm and 10 µm.

to compare the 2D and 3D FEM simulations, the same boundary conditions are applied. The thermal properties and thicknesses of the materials are given in Table II. When performing simulations, the number of mesh elements is important, especially for the smallest dimension of the studied object, in order to avoid numerical error. Locally-refined mesh is considered in the pads, and in the wire of course. To provide a qualitative idea of the results, Figs. 7(a) and 7(b) present the temperature fields in the two FEM configurations (2D and 3D). In 2D FEM, there is only the heater of width $2b$ on top of the substrate while in 3D FEM there are the conducting pads and the volumic heater of length $L$, width $2b$ and thickness $e$. The results have been obtained in the static regime. To analyse more quantitatively the differences between the configurations, we study numerically the ratio of the thermal conductances $G_{2D}/G_{3D}$ as a function of the heater length over width $L/2b$. We observe in Fig. 7(b) that when the length of the wire is small, the temperature rise through the heater is affected by the device pads and characteristic sizes. Therefore, long wires are required. The red curve in Fig. 7(c) corresponds to the case where the pads are connected to the wire, but they are thermally insulating on their top surfaces, neglecting the impact of the connections to the electrical circuit. Thus the effect that can be observed is mostly the heat dissipation in 3D in the substrate. As can be seen, when the {length over width} aspect ratio is smaller than 150, the thermal conductances of the 2D and 3D geometries do not have the same value. Thus, a real experiment would be considered as 2D only when the aspect ratio is larger than 150.

However, to perform the measurement with such devices, the pads are actually required to be connected to bonding wires to perform the electrical measurements (or to probe tips that would act in the same way). Thus the impact of these bonding wires should be taken into account in 3D FEM simulations. The influence of the bonding wire on thermal conductance is represented by a boundary condition on the top of each pad involving a thermal conductance $G_{th} = \frac{\kappa S}{L}$ (with $\kappa_{gold}$) on a disc $S = \pi R^2$, with $R = 50$ µm and $L = 1.5$ cm (typical length of the bonding wire that conducts the electrical current). The bonding wires are supposed to be made of gold, with thermal conductivity $\kappa_{gold} = 318$ W.m$^{-1}$.K$^{-1}$. The results are shown by the blue curve in Fig. 7(c), where it is clear that the

<table>
<thead>
<tr>
<th>Material</th>
<th>$C$ [J.Kg$^{-1}$.K$^{-1}$]</th>
<th>$\kappa$ [W.m$^{-1}$.K$^{-1}$]</th>
<th>$D$ [$\times10^{-11}$ m$^2$.s$^{-1}$]</th>
<th>$d$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon</td>
<td>710</td>
<td>149</td>
<td>8.80</td>
<td>0.3</td>
</tr>
<tr>
<td>Gold</td>
<td>129</td>
<td>318</td>
<td>12.77</td>
<td>1.8</td>
</tr>
<tr>
<td>Plastic</td>
<td>1470</td>
<td>0.18</td>
<td>0.01</td>
<td>3</td>
</tr>
</tbody>
</table>
FIG. 7. Comparison between 2D and 3D FEM simulations for silicon. (a) 2D and (b) top view 3D FEM–simulated profile around a Joule–heated metallic wire on top of the substrate. The squared pads of side $L = 200 \mu m$ are deposited on top of silicon and connected or not to heat baths assimilated to discs of radius $50 \mu m$ representing bonding gold wires of length $L = 1.5$ cm. (c) Impact of the wire shape parameters on the thermal conductance. All the results have been obtained in the static regime.

FIG. 8. Accounting for the total geometry. (a) 3D design. (b) Schematic illustration of the multilayer of our system with the boundary conditions.
wire bonding induces a second effect and that further length is required to satisfy the 2D condition. A length-to-width ratio about 600 is needed with the considered parameters. The highlighted ratios can be difficult to meet experimentally. In this case, full 3D simulations are required to analyze the experiments. Note that the bonding wire conductances are estimated here from heat diffusion as the measurements are usually performed in vacuum. In ambient environment, one would need to compute these conductances as thermal fins and the \{length over width\} ratio would be larger than 600.

**B. Effect of the packaging**

The thermal penetration depth at low frequencies can exceed silicon substrate thicknesses. For instance, at 5 Hz, heat diffusion in silicon takes place over a distance of the order of 1.69 mm, which is greater than most of substrate standard thicknesses. As a result, interfaces and additional layers of our system can also affect the evolution of the temperature oscillations. Fig. 8 presents the full geometry of a typical experimental system. The substrate of thickness 300 µm (layer 1) is placed over a printed circuit board (PCB, thermal conductivity similar to a plastic) of thickness 1.8 mm (layer 2) over a cold finger (element of a cryostat, in which usual 3\(\omega\) experiment are performed) of thickness 3 mm (layer 3) similar to that used in an experiment.\(^{36}\)

The temperature field is shown in Fig. 9. It is confirmed that the impact of the pads is strong, as they are mostly isothermal at the ambient temperature (Figs. 9(a) and 9(c)) and influence significantly

![Temperature field and pads](image-url)
the temperature field in the wire (Fig. 9(b)). Note in particular the effect of the inner (voltage) pads. Here, in contrast to Figs. 7(a) and 7(b), the sample appears to be heated in a large volume and the temperature rise decays slowly on the sides of the wire. In 2D FEM calculations associated to the multilayer, we find that the thermal conductance is $7.4 \times 10^{-3} \text{W.K}^{-1}$ while in DC 3D FEM its value is $4.6 \times 10^{-3} \text{W.K}^{-1}$. This is in agreement with the fact that 3D spreading and the pads play a larger role. Both these values are larger than the 2D one consisting of the silicon substrate with fixed temperature as presented in Section II B 2. These results underline therefore that it is important to consider the full geometry if the sample thickness is too small in comparison to the thermal penetration length.

V. CONCLUSIONS

In conclusion, analytical and numerical studies on diffusive heat conduction in 2D and 3D configurations have illustrated the limitations of the usual analytical model of the $3\omega$ method. We have showed that the effect of the oxide layer on the heat dissipation becomes larger as the heat source becomes narrower, which emphasizes that the wire width is key to the determination of heat dissipation. An error of a factor 2 can be done for a width of 100 nm due to the native oxide. For larger widths, analyzes of electrically-conducting materials in the low-frequency regime should also consider the insulating layers. Similar effects are played by thermal boundary resistances, but it seems that their impacts can be discarded due to the usually-low metal-dielectric resistances. In contrast, the shape of wire is important and some constraints on its sizes have been highlighted. First, its thickness should not exceed 1/20th of its width. Second, a factor on the order $10^3$ between the wire length and width is recommended in order to avoid the side effects due to 3D heat spreading on the one hand and connecting pads on the other hand. Finally, the impact of the packaging below the sample has been underlined, as the frequency range of operation is usually moderate in the $3\omega$ method and it is not always possible to avoid being sensitive to elements below the sample of interest. While many points raised induce strong requirements, it may not be possible experimentally to satisfy all of them. In this case, these results can provide clues about the deviation to ideality. They also show that full 3D simulation may be a solution in complex configurations. We note that no phonon confinement effects was included in the present work, but that it can be added by considering modified (‘effective’) values of the thermal boundary resistances and thermal conductivities.

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